

# CSC D70: Compiler Optimization Dataflow Analysis

Prof. Gennady Pekhimenko

University of Toronto

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*The content of this lecture is adapted from the lectures of  
Todd Mowry and Phillip Gibbons*

# Refreshing from Last Lecture

- Basic Block Formation
- Value Numbering

# Partitioning into Basic Blocks

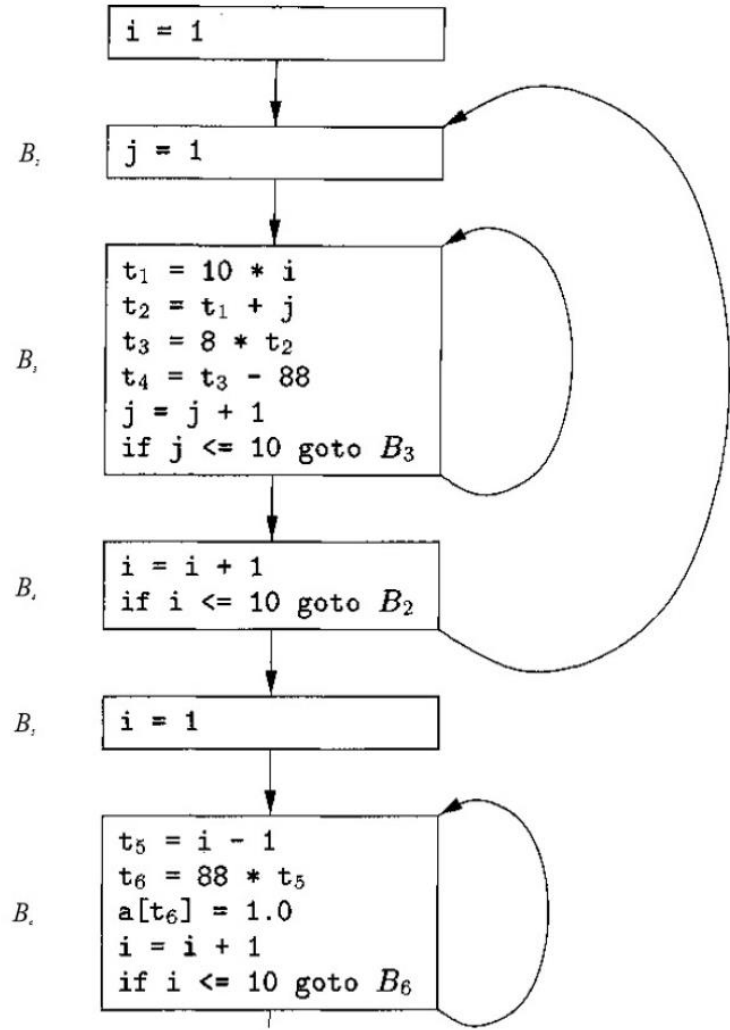
- Identify the leader of each basic block
  - First instruction
  - Any target of a jump
  - Any instruction immediately following a jump
- Basic block starts at leader & ends at instruction immediately before a leader (or the last instruction)

```

★ 1) i = 1
★ 2) j = 1
★ 3) t1 = 10 * i
4) t2 = t1 + j
5) t3 = 8 * t2
6) t4 = t3 - 88
7) a[t4] = 0.0
8) j = j + 1
9) if j <= 10 goto (3)
★ 10) i = i + 1
11) if i <= 10 goto (2)
★ 12) i = 1
★ 13) t5 = i - 1
14) t6 = 88 * t5
15) a[t6] = 1.0
16) i = i + 1
17) if i <= 10 goto (13)

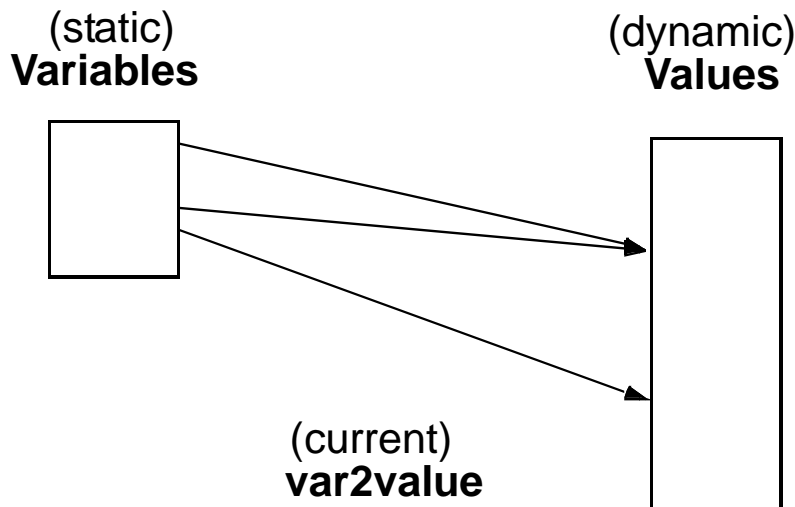
```

★ = Leader



# Value Numbering (VN)

- More explicit with respect to VALUES, and TIME



- each value has its own “number”
    - common subexpression means same value number
  - var2value: current map of variable to value
    - used to determine the value number of current expression
- $r1 + r2 \Rightarrow \text{var2value}(r1) + \text{var2value}(r2)$**

# Algorithm

Data structure:

```
VALUES = Table of
  expression    //[OP, valnum1, valnum2]
  var           //name of variable currently holding expression
```

For each instruction (dst = src1 OP src2) in execution order

```
valnum1 = var2value(src1); valnum2 = var2value(src2);
```

```
IF [OP, valnum1, valnum2] is in VALUES
```

```
  v = the index of expression
```

```
  Replace instruction with CPY dst = VALUES[v].var
```

```
ELSE
```

```
  Add
```

```
    expression = [OP, valnum1, valnum2]
```

```
    var        = dst
```

```
  to VALUES
```

```
  v = index of new entry; tv is new temporary for v
```

```
  Replace instruction with: tv = VALUES[valnum1].var OP VALUES[valnum2].var
```

```
    dst = tv;
```

```
set_var2value (dst, v)
```

# VN Example

Assign: a->r1, b->r2, c->r3, d->r4

a = b+c;           ADD t1 = r2, r3

CPY r1 = t1

b = a-d;           SUB t2 = r1, r4

CPY r2 = t2

c = b+c;           ADD t3 = r2, r3

CPY r3 = t3

d = a-d;           SUB t4 = r1, r4

CPY r4 = t4

# Questions about Assignment #1

- Tutorial #1
- Tutorial #2 next week
  - More in-depth LLVM coverage



# Outline

1. Structure of data flow analysis
2. Example 1: Reaching definition analysis
3. Example 2: Liveness analysis
4. Generalization

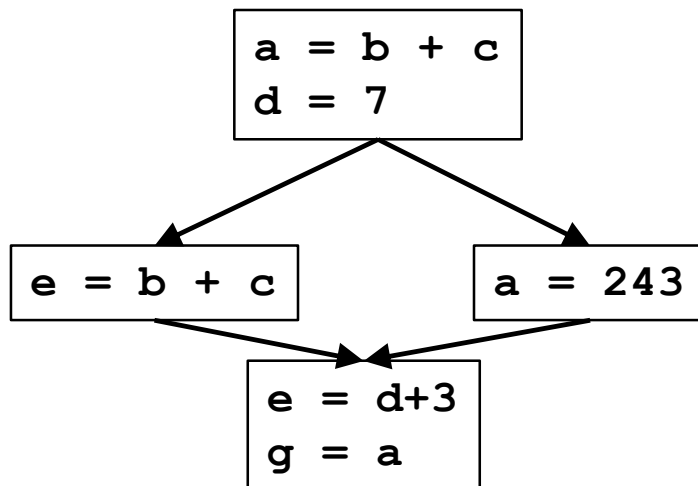
# What is Data Flow Analysis?

- **Local analysis (e.g. value numbering)**
  - analyze effect of each instruction
  - compose effects of instructions to derive information from beginning of basic block to each instruction
- **Data flow analysis**
  - analyze effect of each basic block
  - compose effects of basic blocks to derive information at basic block boundaries
  - from basic block boundaries, apply local technique to generate information on instructions

# What is Data Flow Analysis? (2)

- **Data flow analysis:**
  - Flow-sensitive: sensitive to the control flow in a function
  - intraprocedural analysis
- **Examples of optimizations:**
  - Constant propagation
  - Common subexpression elimination
  - Dead code elimination

# What is Data Flow Analysis? (3)



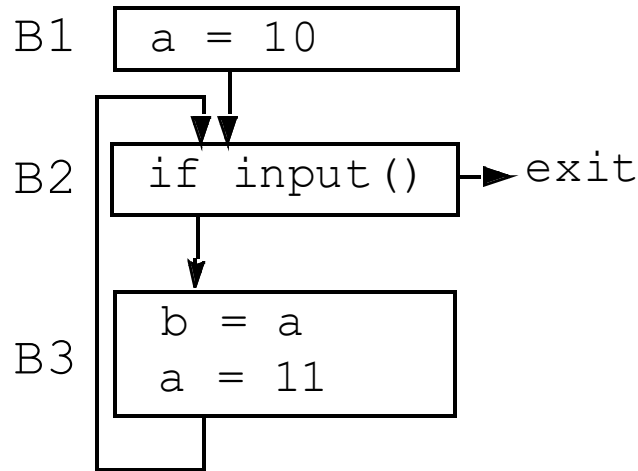
For each variable  $x$  determine:

Value of  $x$ ?

Which “definition” defines  $x$ ?

Is the definition still meaningful (live)?

# Static Program vs. Dynamic Execution



- **Statically:** Finite program
- **Dynamically:** Can have infinitely many possible execution paths
- **Data flow analysis abstraction:**
  - For each point in the program:  
combines information of all the instances of the same program point.
- **Example of a data flow question:**
  - Which definition defines the value used in statement “`b = a`”?

# Effects of a Basic Block

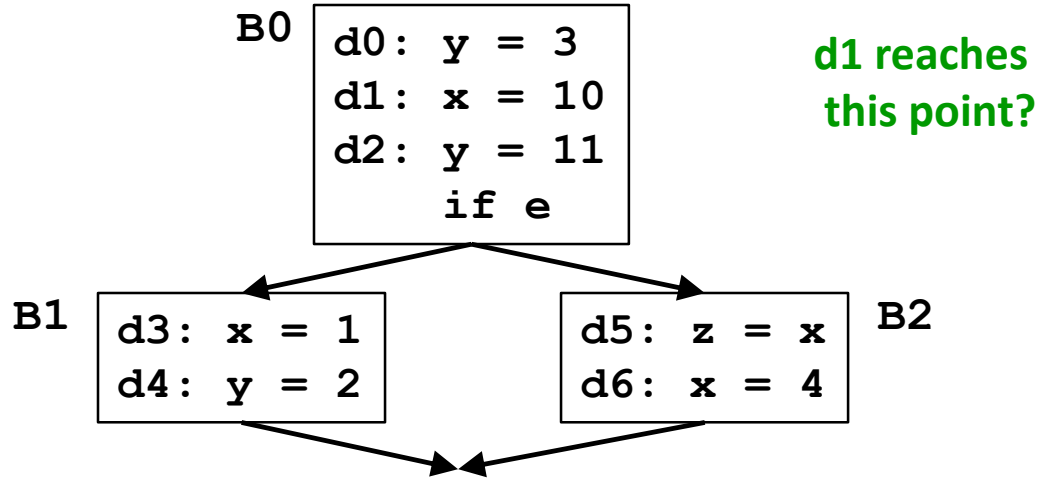
- Effect of a statement:  $a = b+c$ 
  - **Use** variables (b, c)
  - **Kills** an old definition (old definition of a)
  - new **definition** (a)
- Compose effects of statements -> Effect of a basic block
  - A **locally exposed use** in a b.b. is a use of a data item which is not preceded in the b.b. by a definition of the data item
  - any definition of a data item in the basic block **kills** all definitions of the same data item reaching the basic block.
  - A **locally available definition** = last definition of data item in b.b.

# Effects of a Basic Block

A **locally available definition** = last definition of data item in b.b.

```
t1 = r1+r2
r2 = t1
t2 = r2+r1
r1 = t2
t3 = r1*r1
r2 = t3
if r2>100 goto L1
```

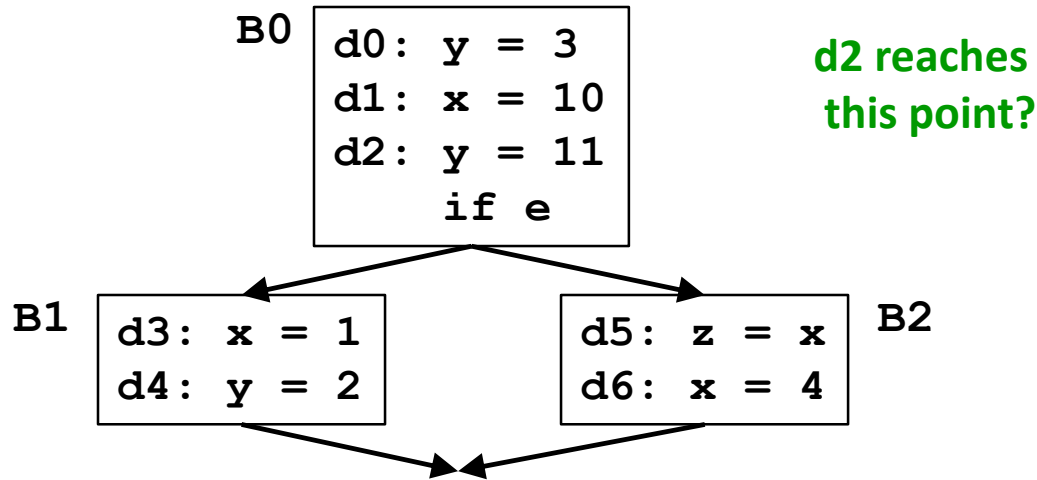
# Reaching Definitions



- Every assignment is a **definition**
- A **definition**  $d$  **reaches** a point  $p$  if **there exists** path from the point immediately following  $d$  to  $p$  such that  $d$  is **not killed** (overwritten) along that path.
- Problem statement
  - For each point in the program, determine if each definition in the program reaches the point
  - A bit vector per program point, vector-length = #defs

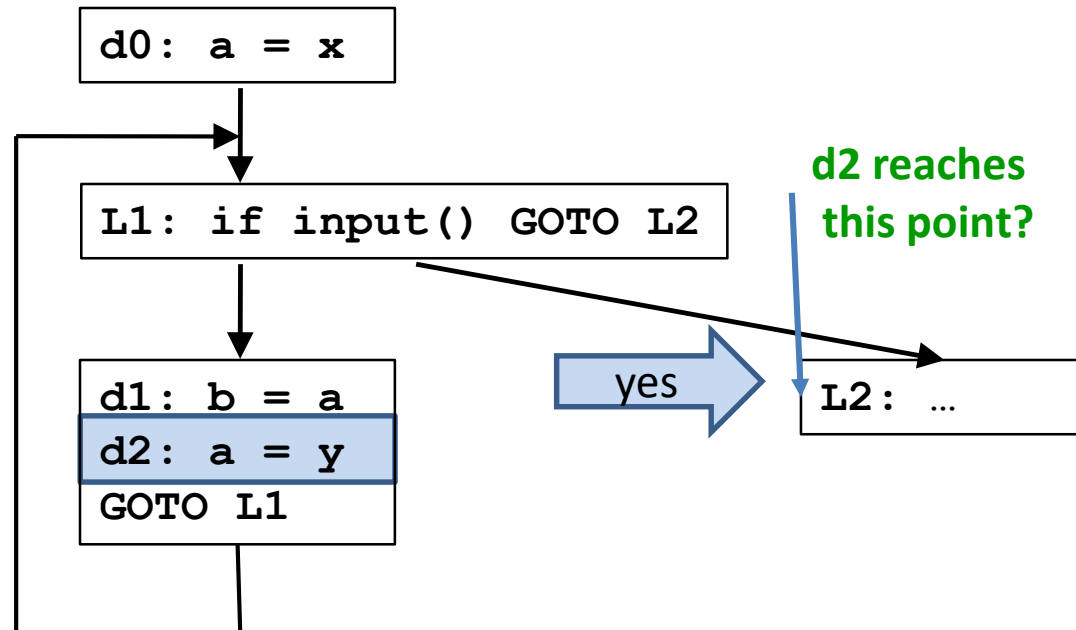


# Reaching Definitions (2)

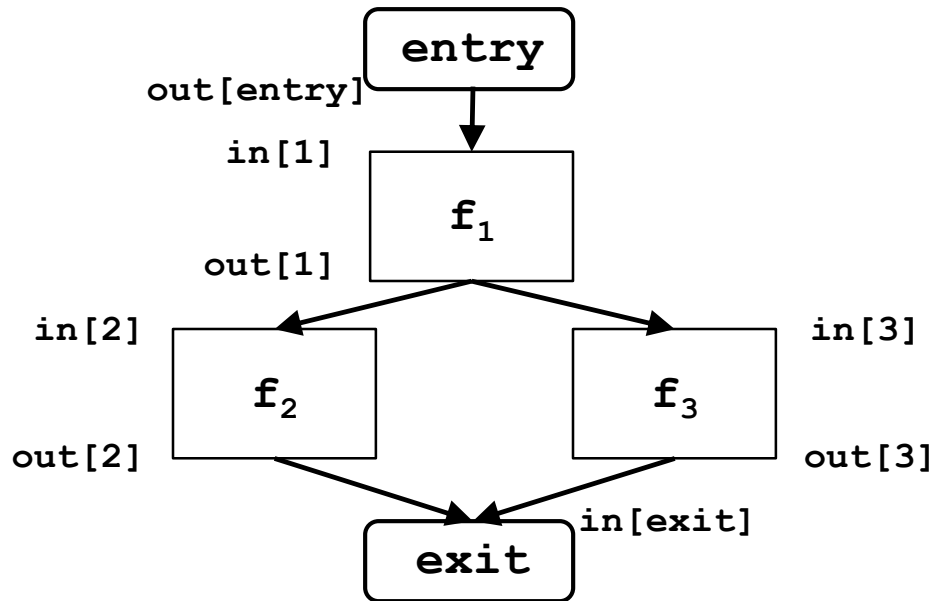


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# Reaching Definitions (3)

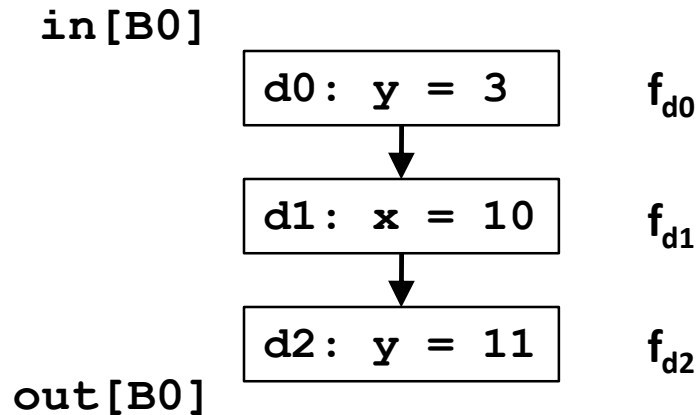


# Data Flow Analysis Schema



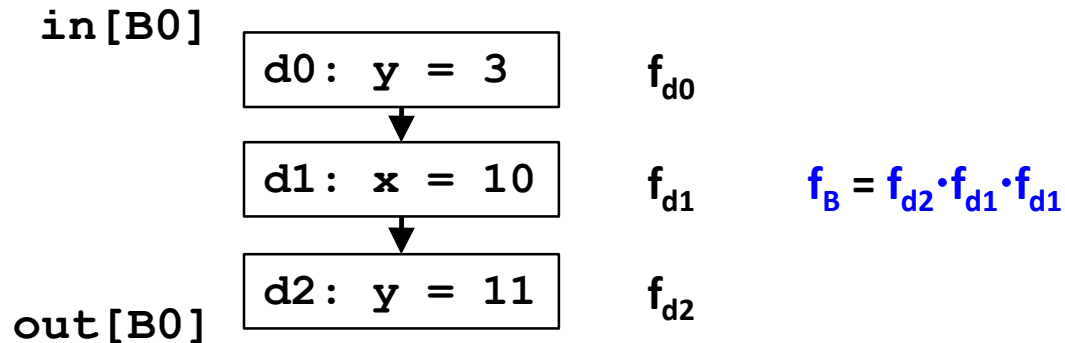
- Build a **flow graph** (nodes = basic blocks, edges = control flow)
- Set up a set of equations between  $in[b]$  and  $out[b]$  for all basic blocks  $b$ 
  - Effect of **code in basic block**:
    - **Transfer function  $f_b$**  relates  $in[b]$  and  $out[b]$ , for same  $b$
  - Effect of **flow of control**:
    - relates  $out[b_1]$ ,  $in[b_2]$  if  $b_1$  and  $b_2$  are **adjacent**
- Find a solution to the equations

# Effects of a Statement



- $f_s$ : A transfer function of a statement
  - abstracts the execution with respect to the problem of interest
- For a statement  $s$  ( $d: x = y + z$ )  
 $out[s] = f_s(in[s]) = Gen[s] \cup (in[s] - Kill[s])$ 
  - **Gen[s]**: definitions generated:  $Gen[s] = \{d\}$
  - **Propagated** definitions:  $in[s] - Kill[s]$ ,  
where **Kill[s]**=set of all other defs to  $x$  in the rest of program

# Effects of a Basic Block



- Transfer function of a statement  $s$ :
  - $out[s] = f_s(in[s]) = Gen[s] \cup (in[s] - Kill[s])$
- Transfer function of a **basic block B**:
  - Composition of transfer functions of statements in B
- $out[B] = f_B(in[B]) = f_{d2} f_{d1} f_{d0}(in[B])$ 

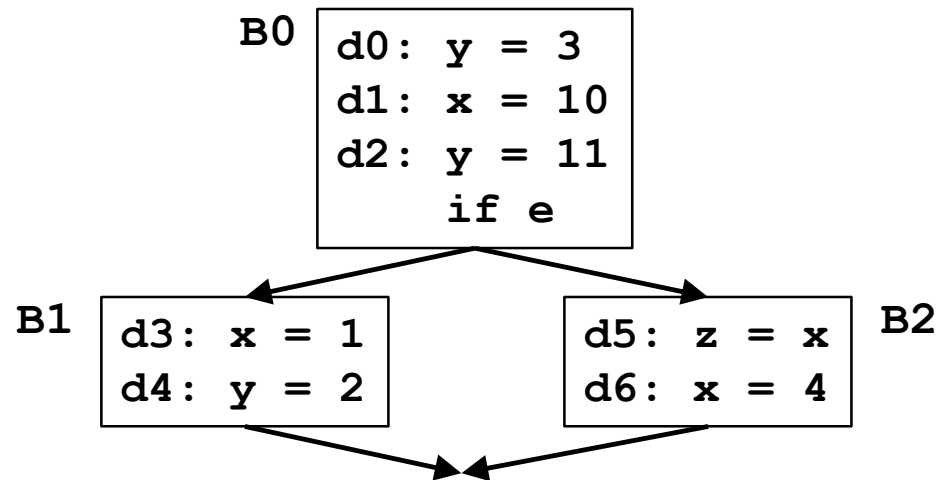
$$= Gen[d_2] \cup (Gen[d_1] \cup (Gen[d_0] \cup (in[B] - Kill[d_0])) - Kill[d_1]) - Kill[d_2]$$

$$= Gen[d_2] \cup (Gen[d_1] \cup (Gen[d_0] - Kill[d_1]) - Kill[d_2]) \cup$$

$$in[B] - (Kill[d_0] \cup Kill[d_1] \cup Kill[d_2])$$

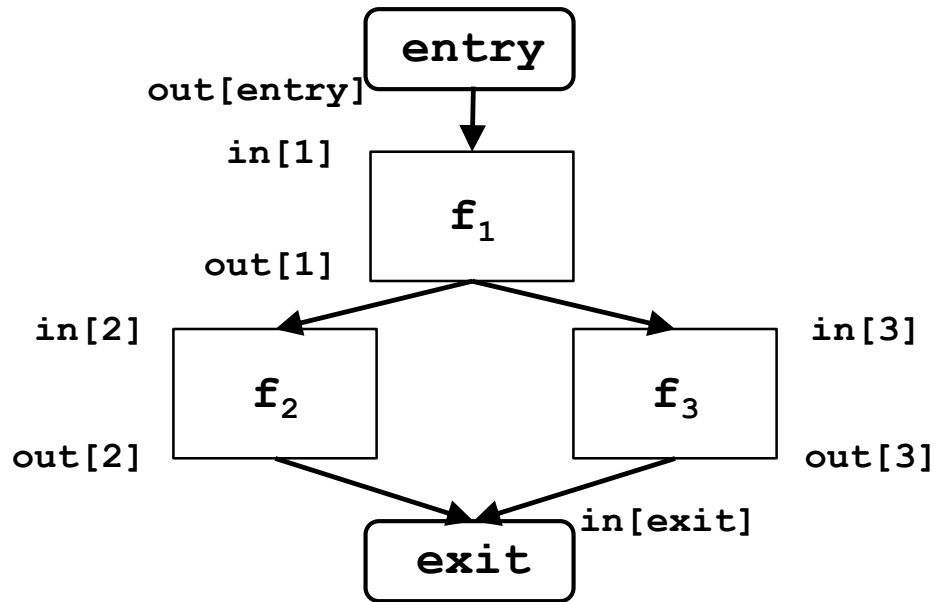
$$= Gen[B] \cup (in[B] - Kill[B])$$
  - $Gen[B]$ : locally exposed definitions (available at end of bb)
  - $Kill[B]$ : set of definitions killed by B

# Example



- a **transfer function**  $f_b$  of a basic block  $b$ :  
 $OUT[b] = f_b(IN[b])$   
incoming reaching definitions  $\rightarrow$  outgoing reaching definitions
- A basic block  $b$ 
  - **generates** definitions:  $Gen[b]$ ,  
– set of locally available definitions in  $b$
  - **kills** definitions:  $in[b] - Kill[b]$ ,  
where  $Kill[b]$  = set of defs (in rest of program) killed by defs in  $b$
- **$out[b] = Gen[b] \cup (in[b] - Kill[b])$**

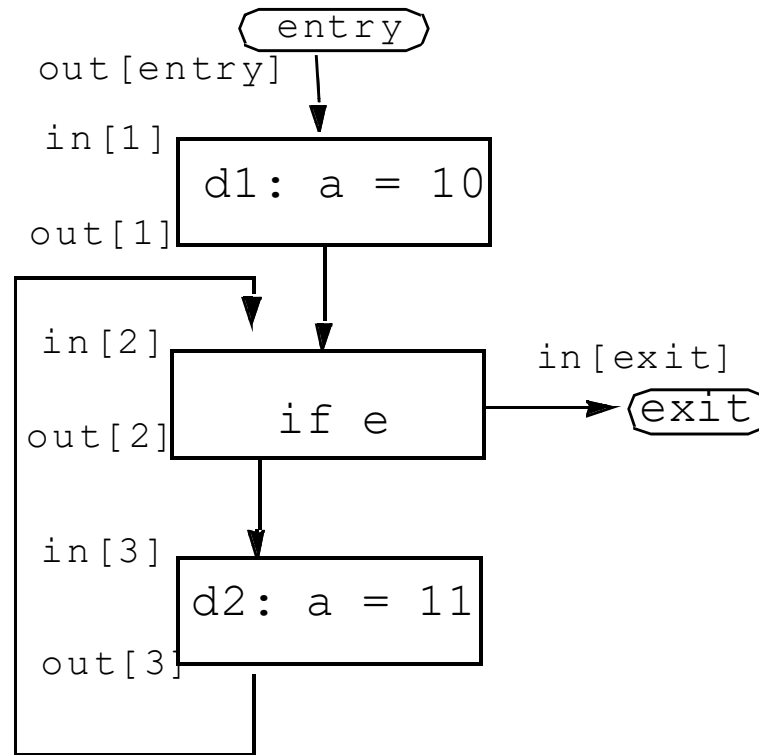
# Effects of the Edges (acyclic)



| f | Gen   | Kill    |
|---|-------|---------|
| 1 | {1,2} | {3,4,6} |
| 2 | {3,4} | {1,2,6} |
| 3 | {5,6} | {1,3}   |

- $out[b] = f_b(in[b])$
- Join node: a node with multiple predecessors
- **meet** operator:  
 $in[b] = out[p_1] \cup out[p_2] \cup \dots \cup out[p_n]$ , where  
 $p_1, \dots, p_n$  are all predecessors of b

# Cyclic Graphs



- Equations still hold
  - $out[b] = f_b(in[b])$
  - $in[b] = out[p_1] \cup out[p_2] \cup \dots \cup out[p_n]$ ,  $p_1, \dots, p_n$  pred.
- Find: fixed point solution



# Reaching Definitions: Iterative Algorithm

```
input: control flow graph CFG = (N, E, Entry, Exit)
```

```
// Boundary condition
```

```
out[Entry] =  $\emptyset$ 
```

```
// Initialization for iterative algorithm
```

```
For each basic block B other than Entry
```

```
out[B] =  $\emptyset$ 
```

```
// iterate
```

```
While (Changes to any out[] occur) {
```

```
For each basic block B other than Entry {
```

```
in[B] =  $\cup$  (out[p]), for all predecessors p of B
```

```
out[B] =  $f_B$ (in[B]) // out[B]=gen[B] $\cup$ (in[B]-kill[B])
```

```
}
```

# Reaching Definitions: Worklist Algorithm

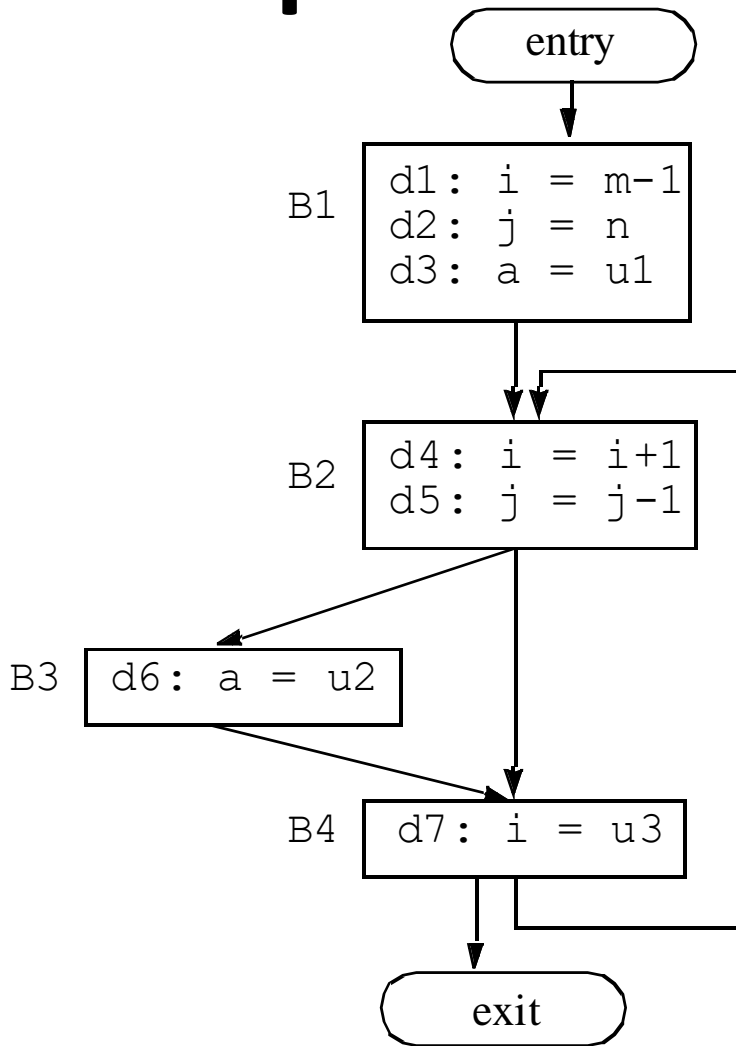
```
input: control flow graph CFG = (N, E, Entry, Exit)

// Initialize
  out[Entry] =  $\emptyset$            // can set out[Entry] to special def
                                // if reaching then undefined use

  For all nodes i
    out[i] =  $\emptyset$            // can optimize by out[i]=gen[i]
  ChangedNodes = N

// iterate
  While ChangedNodes  $\neq \emptyset$  {
    Remove i from ChangedNodes
    in[i] = U (out[p]), for all predecessors p of i
    oldout = out[i]
    out[i] =  $f_i$ (in[i])         // out[i]=gen[i]U(in[i]-kill[i])
    if (oldout  $\neq$  out[i]) {
      for all successors s of i
        add s to ChangedNodes
    }
  }
}
```

# Example



|          | First Pass | Second Pass |
|----------|------------|-------------|
| IN[B1]   | 000 00 0 0 | 000 00 0 0  |
| OUT[B1]  | 111 00 0 0 | 111 00 0 0  |
| IN[B2]   | 111 00 0 0 | 111 01 1 1  |
| OUT[B2]  | 001 11 0 0 | 001 11 1 0  |
| IN[B3]   | 001 11 0 0 | 001 11 1 0  |
| OUT[B3]  | 000 11 1 0 | 000 11 1 0  |
| IN[B4]   | 001 11 1 0 | 001 11 1 0  |
| OUT[B4]  | 001 01 1 1 | 001 01 1 1  |
| IN[exit] | 001 01 1 1 | 001 01 1 1  |

# Live Variable Analysis

- **Definition**

- A variable  $v$  is **live** at point  $p$  if
  - the value of  $v$  is used along some path in the flow graph starting at  $p$ .
- Otherwise, the variable is **dead**.

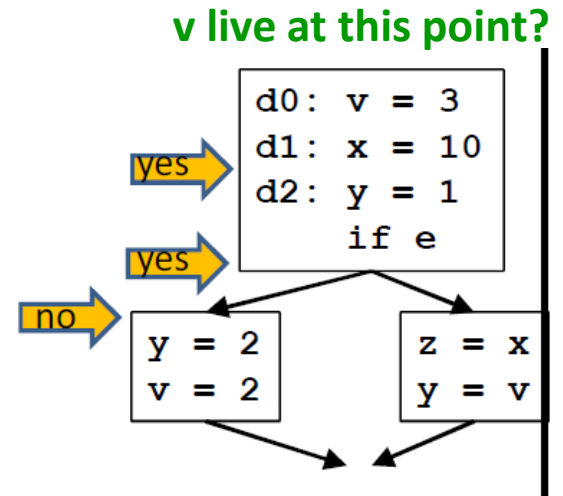
- **Motivation**

- e.g. register allocation

```
for i = 0 to n
  ... i ...
...
for i = 0 to n
  ... i ...
```

- **Problem statement**

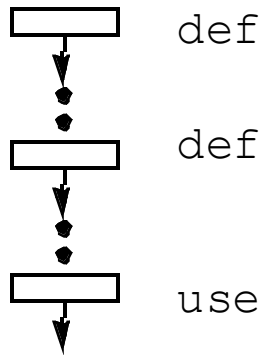
- For each basic block
  - determine if each variable is live in each basic block
- Size of bit vector: one bit for each variable



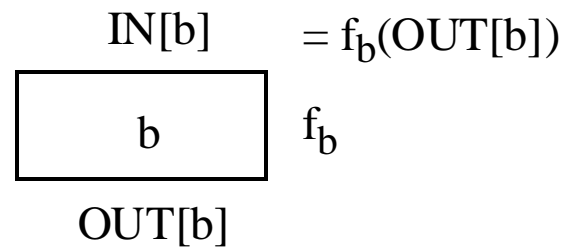
# Transfer Function

- Insight: Trace uses backwards to the definitions

an execution path



control flow



example

d3: a = 1

d4: b = 1

d5: c = a

d6: a = 4

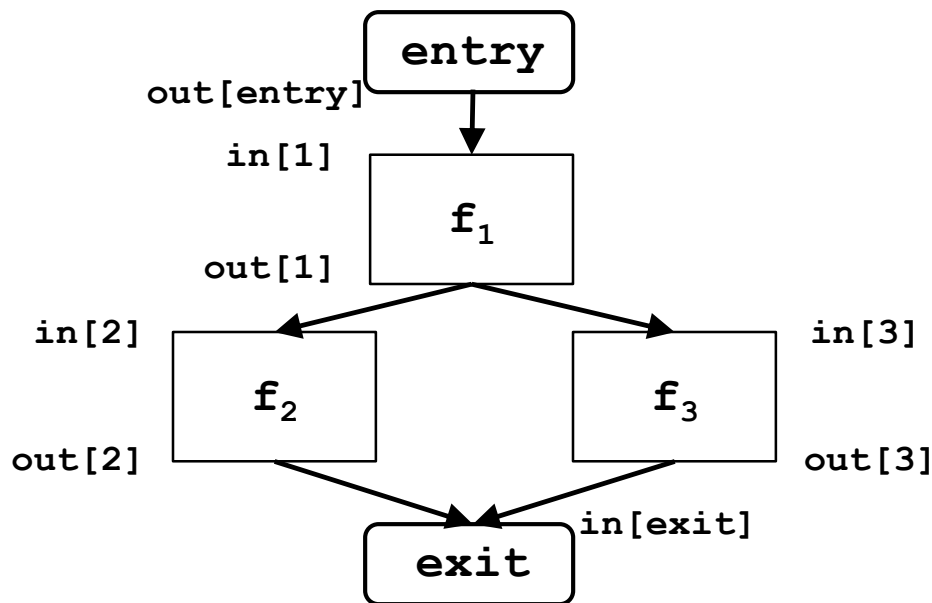
- A basic block **b** can

- generate live variables: **Use[b]**
  - set of locally exposed uses in b
- propagate incoming live variables: **OUT[b] - Def[b]**,
  - where **Def[b]** = set of variables defined in b.b.

- transfer function** for block b:

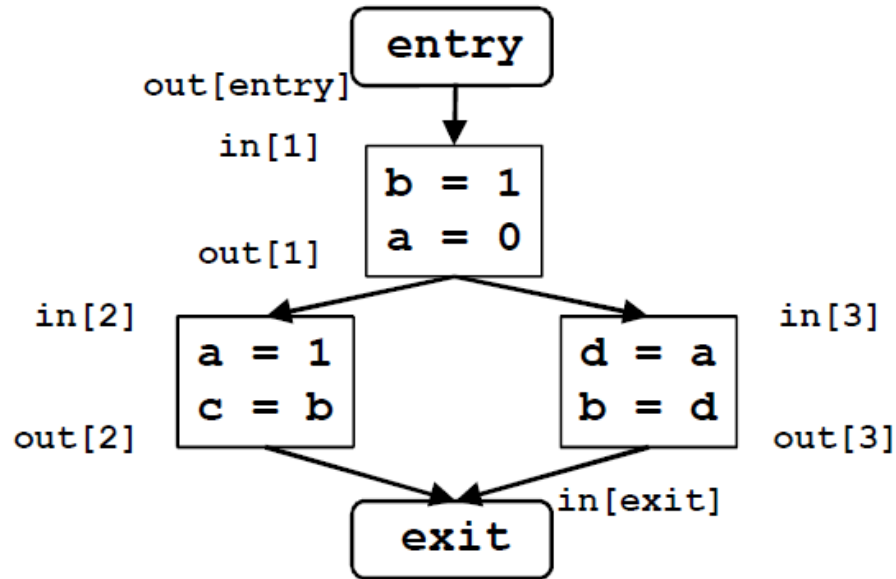
$$\text{in}[b] = \text{Use}[b] \cup (\text{out}(b) - \text{Def}[b])$$

# Flow Graph



- $in[b] = f_b(out[b])$
- **Join node**: a node with multiple **successors**
- **meet** operator:  
 $out[b] = in[s_1] \cup in[s_2] \cup \dots \cup in[s_n]$ , where  
 $s_1, \dots, s_n$  are all successors of  $b$

# Flow Graph (2)



| f | Use | Def   |
|---|-----|-------|
| 1 | {}  | {a,b} |
| 2 | {b} | {a,c} |
| 3 | {a} | {b,d} |

- $in[b] = f_b(out[b])$
- **Join node**: a node with multiple successors
- **meet** operator:  
 $out[b] = in[s_1] \cup in[s_2] \cup \dots \cup in[s_n]$ , where  
 $s_1, \dots, s_n$  are all successors of b

# Liveness: Iterative Algorithm

```
input: control flow graph CFG = (N, E, Entry, Exit)
```

```
// Boundary condition
```

```
in[Exit] =  $\emptyset$ 
```

```
// Initialization for iterative algorithm
```

```
For each basic block B other than Exit
```

```
in[B] =  $\emptyset$ 
```

```
// iterate
```

```
While (Changes to any in[] occur) {
```

```
For each basic block B other than Exit {
```

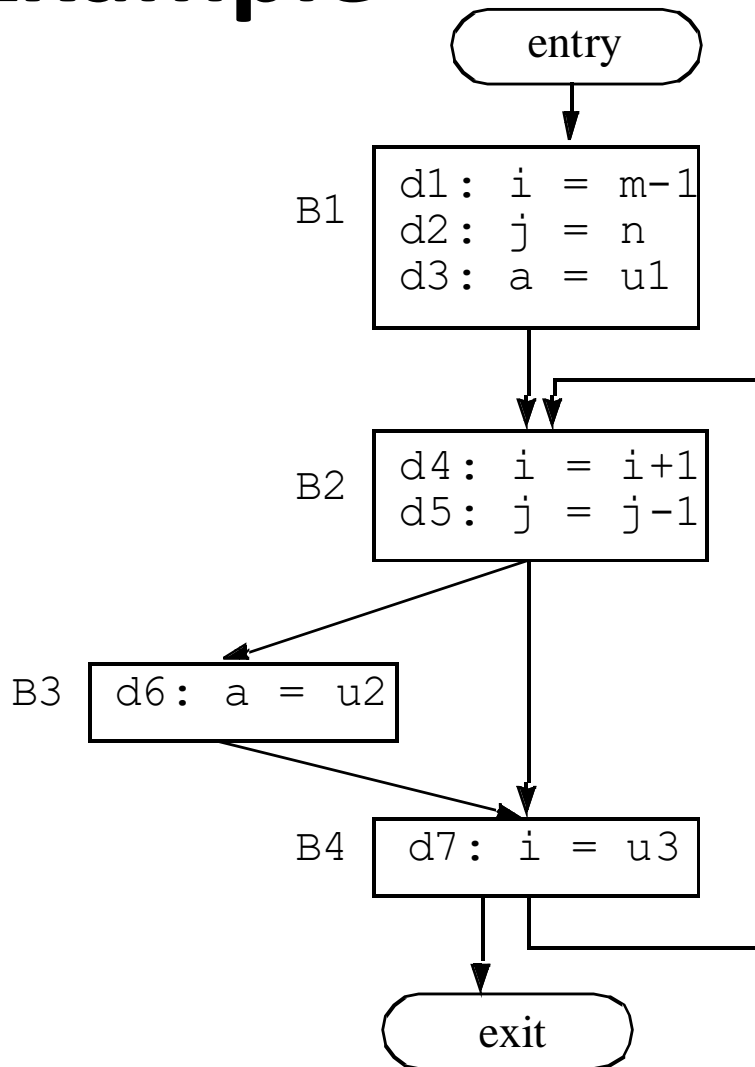
```
out[B] =  $\cup$  (in[s]), for all successors s of B
```

```
in[B] =  $f_B$ (out[B]) // in[B]=Use[B] $\cup$ (out[B]-Def[B])
```

```
}
```



# Example



|            | First Pass     | Second Pass    |
|------------|----------------|----------------|
| OUT[entry] | {m,n,u1,u2,u3} | {m,n,u1,u2,u3} |
| IN[B1]     | {m,n,u1,u2,u3} | {m,n,u1,u2,u3} |
| OUT[B1]    | {i,j,u2,u3}    | {i,j,u2,u3}    |
| IN[B2]     | {i,j,u2,u3}    | {i,j,u2,u3}    |
| OUT[B2]    | {u2,u3}        | {j,u2,u3}      |
| IN[B3]     | {u2,u3}        | {j,u2,u3}      |
| OUT[B3]    | {u3}           | {j,u2,u3}      |
| IN[B4]     | {u3}           | {j,u2,u3}      |
| OUT[B4]    | {}             | {i,j,u2,u3}    |

# Framework

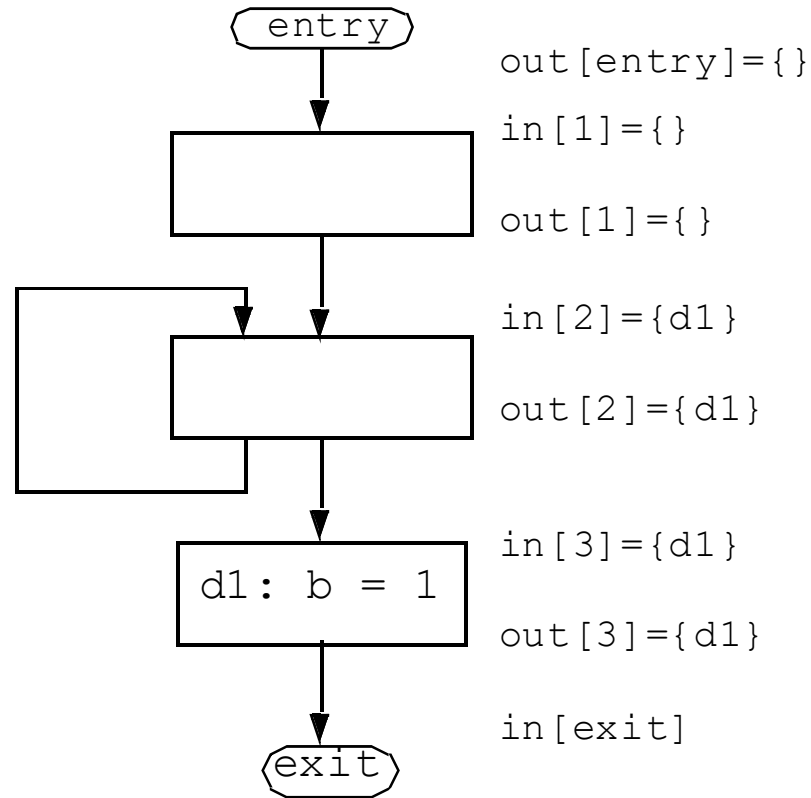
|                             | Reaching Definitions   | Live Variables  |
|-----------------------------|--|---|
| Domain                      | Sets of definitions  | Sets of variables   |
| Direction                   | forward:<br>$out[b] = f_b(in[b])$<br>$in[b] = \wedge out[pred(b)]$ | backward:<br>$in[b] = f_b(out[b])$<br>$out[b] = \wedge in[succ(b)]$ |
| Transfer function           | $f_b(x) = Gen_b \cup (x - Kill_b)$                                 | $f_b(x) = Use_b \cup (x - Def_b)$                                   |
| Meet Operation ( $\wedge$ ) | $\cup$   | $\cup$  |
| Boundary Condition          | $out[entry] = \emptyset$   | $in[exit] = \emptyset$  |
| Initial interior points     | $out[b] = \emptyset$   | $in[b] = \emptyset$   |

Other examples (e.g., Available expressions), defined in ALSU 9.2.6

# Thought Problem 1. “Must-Reach” Definitions

- **A definition  $D$  ( $a = b+c$ ) must reach point  $P$  iff**
  - $D$  appears at least once along on all paths leading to  $P$
  - $a$  is not redefined along any path after last appearance of  $D$  and before  $P$
- **How do we formulate the data flow algorithm for this problem?**

# Problem 2: A legal solution to (May) Reaching Def?



- Will the worklist algorithm generate this answer?

# Questions

- **Correctness**
  - equations are satisfied, if the program terminates.
- **Precision: how good is the answer?**
  - is the answer ONLY a union of all possible executions?
- **Convergence: will the analysis terminate?**
  - or, will there always be some nodes that change?
- **Speed: how fast is the convergence?**
  - how many times will we visit each node?

# Foundations of Data Flow Analysis

- 1. Meet operator**
- 2. Transfer functions**
- 3. Correctness, Precision, Convergence**
- 4. Efficiency**

- Reference: ALSU pp. 613-631
- Background: Hecht and Ullman, Kildall, Allen and Cocke[76]
- Marlowe & Ryder, Properties of data flow frameworks: a unified model. Rutgers tech report, Apr. 1988

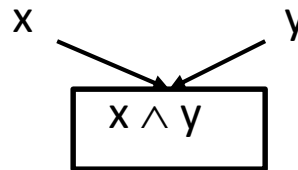
# A Unified Framework

- **Data flow problems are defined by**
  - Domain of values:  $V$
  - Meet operator ( $V \wedge V \rightarrow V$ ), initial value
  - A set of transfer functions ( $V \rightarrow V$ )
- **Usefulness of unified framework**
  - To answer questions such as **correctness, precision, convergence, speed of convergence** for a family of problems
    - If meet operators and transfer functions have properties  $X$ , then we know  $Y$  about the above.
  - Reuse code

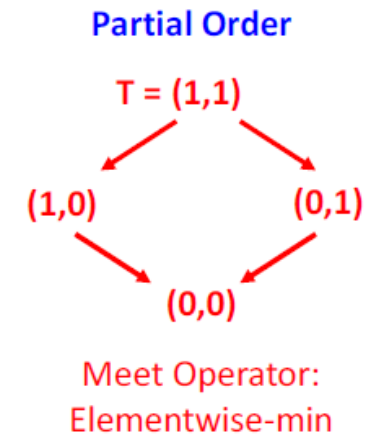
# Meet Operator

- **Properties of the meet operator**

- **commutative:**  $x \wedge y = y \wedge x$



- **idempotent:**  $x \wedge x = x$
- **associative:**  $x \wedge (y \wedge z) = (x \wedge y) \wedge z$
- there is a **Top** element **T** such that  $x \wedge T = x$



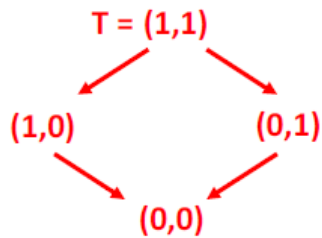
- **Meet operator defines a partial ordering on values**

- $x \leq y$  if and only if  $x \wedge y = x$  ( **$y \rightarrow x$  in diagram**)
  - **Transitivity:** if  $x \leq y$  and  $y \leq z$  then  $x \leq z$
  - **Antisymmetry:** if  $x \leq y$  and  $y \leq x$  then  $x = y$
  - **Reflexivity:**  $x \leq x$

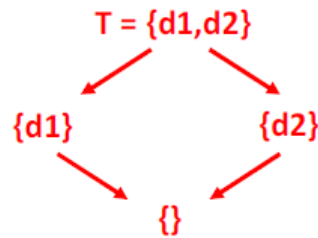


# Partial Order

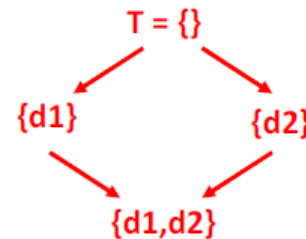
- Example: let  $V = \{x \mid \text{such that } x \subseteq \{d_1, d_2\}\}$ ,  $\wedge = \cap$



Meet Operator:  
Elementwise-min



Meet Operator:  
Intersection



Meet Operator:  
Union

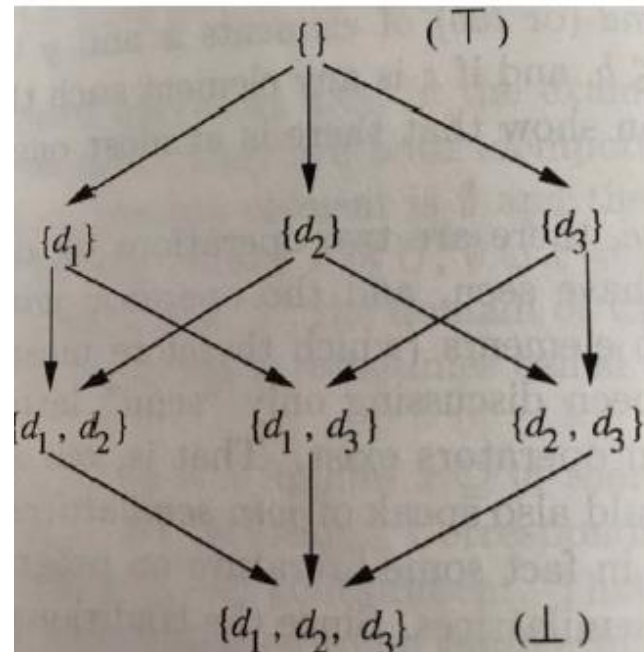
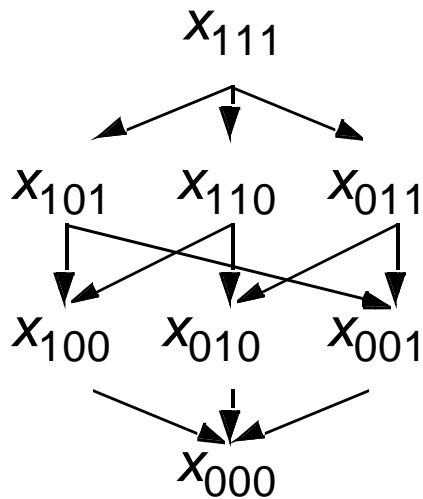
- Top and Bottom elements
  - Top  $T$  such that:  $x \wedge T = x$
  - Bottom  $\perp$  such that:  $x \wedge \perp = \perp$
- Values and meet operator in a data flow problem define a semi-lattice:
  - there exists a  $T$ , but not necessarily a  $\perp$ .
- $x, y$  are ordered:  $x \leq y$  then  $x \wedge y = x$  ( $y \rightarrow x$  in diagram)
- what if  $x$  and  $y$  are not ordered?
  - $x \wedge y \leq x$ ,  $x \wedge y \leq y$ , and if  $w \leq x$ ,  $w \leq y$ , then  $w \leq x \wedge y$

# One vs. All Variables/Definitions

- Lattice for each variable: e.g. intersection



- Lattice for three variables:



# Descending Chain

- **Definition**

- The **height** of a lattice is the largest number of **> relations** that will fit in a descending chain.

$$x_0 > x_1 > x_2 > \dots$$

- **Height of values in reaching definitions?**

- **Important property: finite descending chain**

- **Can an infinite lattice have a finite descending chain? yes**

- **Example: Constant Propagation/Folding**

- To determine if a variable is a constant

- **Data values**

- undef, ... -1, 0, 1, 2, ..., not-a-constant

# Transfer Functions

- **Basic Properties**  $f: V \rightarrow V$ 
  - Has an identity function
    - There exists an  $f$  such that  $f(x) = x$ , for all  $x$ .
  - Closed under composition
    - if  $f_1, f_2 \in F$ , then  $f_1 \cdot f_2 \in F$

# Monotonicity

- A framework  $(F, V, \wedge)$  is **monotone** if and only if
  - $x \leq y$  implies  $f(x) \leq f(y)$
  - i.e. a “smaller or equal” input to the same function will always give a “smaller or equal” output
- **Equivalently**, a framework  $(F, V, \wedge)$  is **monotone** if and only if
  - $f(x \wedge y) \leq f(x) \wedge f(y)$
  - i.e. merge input, then apply  $f$  is **small than or equal to** apply the transfer function individually and then merge the result

# Example

- Reaching definitions:  $f(x) = \text{Gen} \cup (x - \text{Kill})$ ,  $\wedge = \cup$

- Definition 1:

- $x_1 \leq x_2$ ,  $\text{Gen} \cup (x_1 - \text{Kill}) \leq \text{Gen} \cup (x_2 - \text{Kill})$

- Definition 2:

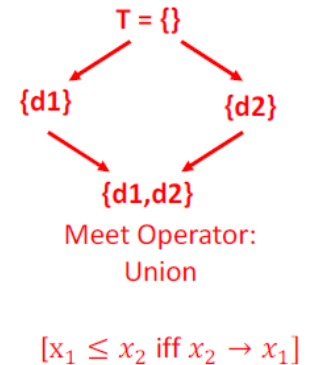
- $(\text{Gen} \cup (x_1 - \text{Kill})) \cup (\text{Gen} \cup (x_2 - \text{Kill}))$   
 $= (\text{Gen} \cup ((x_1 \cup x_2) - \text{Kill}))$

- **Note: Monotone framework does not mean that  $f(x) \leq x$**

- e.g., reaching definition for two definitions in program
- suppose:  $f_x: \text{Gen}_x = \{d_1, d_2\}$ ;  $\text{Kill}_x = \{\}$

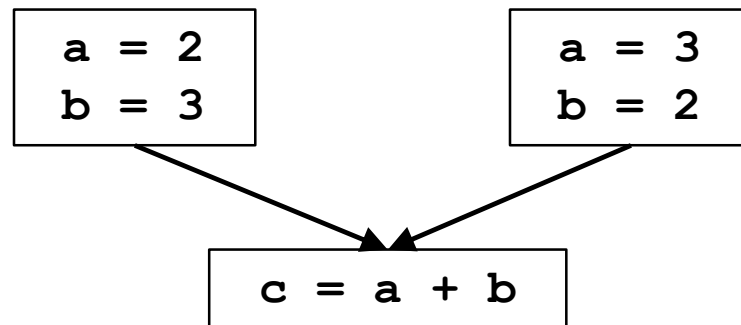
- **If input(second iteration)  $\leq$  input(first iteration)**

- result(second iteration)  $\leq$  result(first iteration)



# Distributivity

- A framework  $(F, V, \wedge)$  is **distributive** if and only if
  - $f(x \wedge y) = f(x) \wedge f(y)$
  - i.e. merge input, then apply  $f$  is **equal to** apply the transfer function individually then merge result
- Example: Constant Propagation is NOT distributive



# Data Flow Analysis

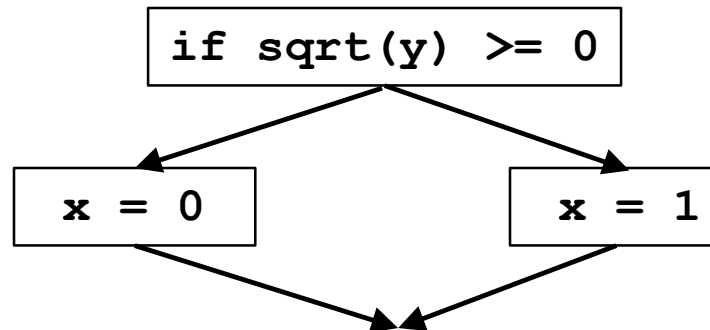
- **Definition**

- Let  $f_1, \dots, f_m : \in F$ , where  $f_i$  is the transfer function for node  $i$ 
  - $f_p = f_{n_k} \cdot \dots \cdot f_{n_1}$ , where  $p$  is a path through nodes  $n_1, \dots, n_k$
  - $f_p = \text{identify function}$ , if  $p$  is an empty path

- **Ideal data flow answer:**

- For each node  $n$ :

$\bigwedge f_{p_i}(T)$ , for all possibly executed paths  $p_i$  reaching  $n$ .



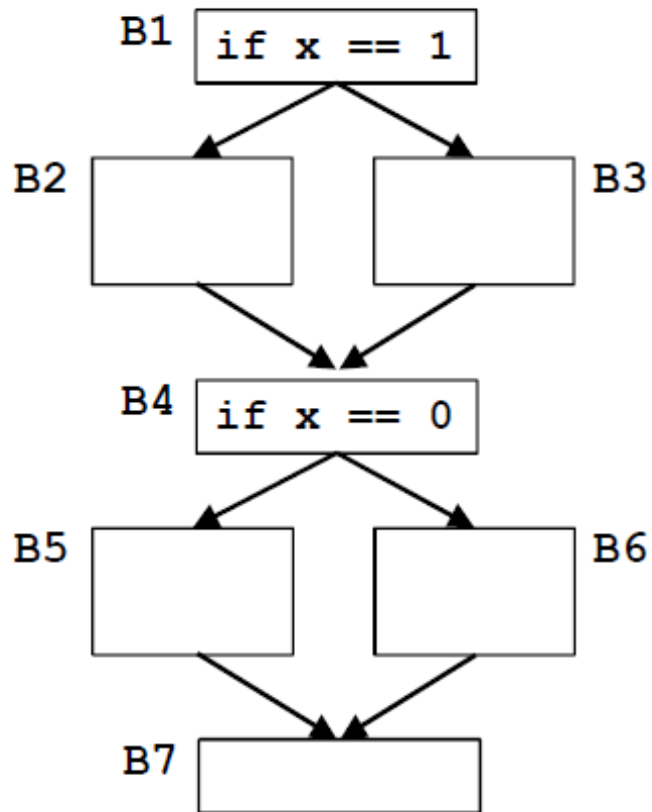
- **But determining all possibly executed paths is undecidable**



# Meet-Over-Paths (MOP)

- Err in the conservative direction
- **Meet-Over-Paths (MOP):**
  - For each node  $n$ :
$$\text{MOP}(n) = \bigwedge f_{p_i}(T), \text{ for all paths } p_i \text{ reaching } n$$
  - a path exists as long there is an edge in the code
  - consider more paths than necessary
  - $\text{MOP} = \text{Perfect-Solution} \wedge \text{Solution-to-Unexecuted-Paths}$
  - $\text{MOP} \leq \text{Perfect-Solution}$
  - Potentially more constrained, solution is small
    - hence *conservative*
  - It is not **safe** to be  $>$  Perfect-Solution!
- **Desirable solution: as close to MOP as possible**

# MOP Example



Assume: B2 & B3 do not update x

Ideal: Considers only 2 paths  
B1-B2-B4-B6-B7 (i.e., x=1)  
B1-B3-B4-B5-B7 (i.e., x=0)

MOP: Also considers unexecuted paths  
B1-B2-B4-B5-B7  
B1-B3-B4-B6-B7

# Solving Data Flow Equations

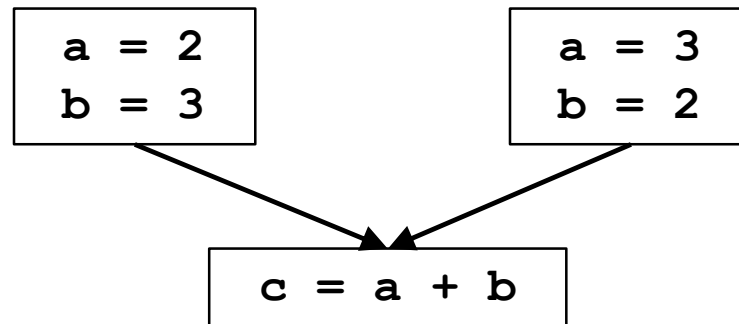
- **Example: Reaching definitions**
  - $\text{out}[\text{entry}] = \{\}$
  - $\text{Values} = \{\text{subsets of definitions}\}$
  - **Meet operator:**  $\cup$ 
    - $\text{in}[b] = \cup \text{out}[p]$ , for all predecessors  $p$  of  $b$
  - **Transfer functions:**  $\text{out}[b] = \text{gen}_b \cup (\text{in}[b] - \text{kill}_b)$
- **Any solution satisfying equations = Fixed Point Solution (FP)**
- **Iterative algorithm**
  - initializes  $\text{out}[b]$  to  $\{\}$
  - if converges, then it computes **Maximum Fixed Point (MFP)**:
    - **MFP is the largest of all solutions to equations**
- **Properties:**
  - $\text{FP} \leq \text{MFP} \leq \text{MOP} \leq \text{Perfect-solution}$
  - FP, MFP are safe
  - $\text{in}(b) \leq \text{MOP}(b)$

# Partial Correctness of Algorithm

- If data flow framework is **monotone**, then if the algorithm converges,  $IN[b] \leq MOP[b]$
- **Proof: Induction on path lengths**
  - Define  $IN[entry] = OUT[entry]$   
and transfer function of entry = Identity function
  - Base case: path of length 0
    - Proper initialization of  $IN[entry]$
  - If true for path of length  $k$ ,  $p_k = (n_1, \dots, n_k)$ , then true for path of length  $k+1$ :  $p_{k+1} = (n_1, \dots, n_{k+1})$ 
    - Assume:  $IN[n_k] \leq f_{n_{k-1}}(f_{n_{k-2}}(\dots f_{n_1}(IN[entry])))$
    - $IN[n_{k+1}] = OUT[n_k] \wedge \dots$ 
      - $\leq OUT[n_k]$
      - $\leq f_{n_k}(IN[n_k])$
      - $\leq f_{n_{k-1}}(f_{n_{k-2}}(\dots f_{n_1}(IN[entry])))$

# Precision

- If data flow framework is **distributive**, then if the algorithm converges,  **$IN[b] = MOP[b]$**



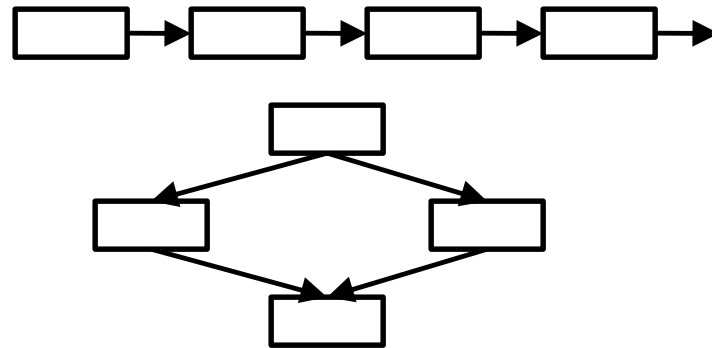
- Monotone but not distributive: behaves as if there are additional paths

# Additional Property to Guarantee Convergence

- Data flow framework (**monotone**) converges if there is a **finite descending chain**
- For each variable  $IN[b]$ ,  $OUT[b]$ , consider the sequence of values set to each variable **across iterations**:
  - if sequence for  $in[b]$  is monotonically decreasing
    - sequence for  $out[b]$  is monotonically decreasing
      - ( $out[b]$  initialized to  $T$ )
  - if sequence for  $out[b]$  is monotonically decreasing
    - sequence of  $in[b]$  is monotonically decreasing

# Speed of Convergence

- Speed of convergence depends on order of node visits



- Reverse “direction” for backward flow problems

# Reverse Postorder

- Step 1: depth-first post order

```
main() {  
    count = 1;  
    Visit(root);  
}  
Visit(n) {  
    for each successor s that has not been  
visited  
        Visit(s);  
    PostOrder(n) = count;  
    count = count+1;  
}
```

- Step 2: reverse order

```
For each node i  
    rPostOrder = NumNodes - PostOrder(i)
```



# Depth-First Iterative Algorithm (forward)

```
input: control flow graph CFG = (N, E, Entry, Exit)

/* Initialize */
  out[entry] = init_value
  For all nodes i
    out[i] = ⊥
  Change = True

/* iterate */
  While Change {
    Change = False
    For each node i in rPostOrder {
      in[i] =  $\wedge$ (out[p]), for all predecessors p of i
      oldout = out[i]
      out[i] =  $f_i$ (in[i])
      if oldout  $\neq$  out[i]
        Change = True
    }
  }
```

# Speed of Convergence

- **If cycles do not add information**
  - information can flow in one pass down a series of nodes of increasing order number:
    - e.g., 1 -> 4 -> 5 -> 7 -> 2 -> 4 ...
  - passes determined by **number of back edges in the path**
    - essentially the nesting depth of the graph
  - **Number of iterations = number of back edges in any acyclic path + 2**
    - (2 are necessary even if there are no cycles)
- **What is the depth?**
  - corresponds to depth of intervals for “reducible” graphs
  - in real programs: average of 2.75

# A Check List for Data Flow Problems

- **Semi-lattice**
  - set of values
  - meet operator
  - top, bottom
  - finite descending chain?
- **Transfer functions**
  - function of each basic block
  - monotone
  - distributive?
- **Algorithm**
  - initialization step (entry/exit, other nodes)
  - visit order: rPostOrder
  - depth of the graph

# Conclusions

- Dataflow analysis examples
  - Reaching definitions
  - Live variables
- Dataflow formation definition
  - Meet operator
  - Transfer functions
  - Correctness, Precision, Convergence
  - Efficiency

# CSC D70: Compiler Optimization Dataflow Analysis

Prof. Gennady Pekhimenko

University of Toronto

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*The content of this lecture is adapted from the lectures of  
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